DESIGN FOR STIFFNESS IN ROTATIONALLY MOULDED PLASTIC PARTS

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Introduction

Polyethylene accounts for the vast majority of the rotationally moulded products manufactured worldwide[1]. This material has attained this dominant position because it exhibits the excellent thermal stability and good melt flow characteristics that are so crucial to the rotational moulding process. However, the main mechanical properties of polyethylene (such as strength and stiffness) are amongst the lowest values available in any thermoplastic[2]. Hence, designers of rotomoulded products must use shape very efficiently to impart stiffness to the moulded article. In some cases, special features such as "kiss-off" points are used to give improved stiffness[1, 3]. However, more generally, Box Ribs are used to improve stiffness per unit weight.

A flat sheet is very effective at resisting axial forces but is very inefficient at resisting transverse forces. Axial stiffness is proportional to the thickness of the sheet and flexural stiffness is proportional to the cube of the thickness. Thus doubling the thickness of the part doubles the axial stiffness and gives an 8 (ie 2^3) times improvement in flexural stiffness. Increasing the thickness of the part is therefore a common way of increasing the stiffness of rotomoulded parts. However, this has a number of major disadvantages – cycle times are prolonged due to the poor thermal conductivity of the plastic and costs are higher due to the extra material used.

In the injection moulding industry, designers utilise ribs to impart stiffness to thin plates[4]. This means that fast cycles can be achieved in the manufacture of stiff and relatively inexpensive parts. Unfortunately the rotational moulding of such sections is very difficult. Hence, in the rotational moulding industry it is common to use Box Ribs to provide "depth". In this way the moulder can effectively increase the thickness of the moulded part whilst keeping weight and cost within acceptable bounds. However, it is not readily apparent what the dimensions of the Box Ribs should be. Deep Box Ribs improve the transverse, or flexural, stiffness but at the expense of the axial stiffness. Clearly, there are many permutations of dimensions that are possible, and this raises the question, "Is there an optimum design configuration?"

Unfortunately, there is no simple answer to this question. There are design guidelines, which give details of shapes that have worked well in the past[5, 6]. Quite often these practical guidelines are not optimised in terms of stiffness because they have to take into account ease of moulding etc.

Consider a corrugated panel as shown in Fig 1. Such Box Ribs are common in rotational moulding and in practice may be subjected to loads that are (a) axial, (b) bending with the loading parallel to the Box Ribs or (c) bending with the loading perpendicular to the Box Ribs[2, 7]. The latter will give the greatest stiffness performance and generally the flexural stiffness will increase dramatically as the depth of the Box Ribs increase. However, for the other two modes of loading, stiffness will decrease as the depth of the Box Ribs increase. As it is seldom that one type of loading occurs in isolation, one always has to accept that the dimensions chosen for the Box Ribs will represent a compromise. Gains made in one direction will generally be at the expense of stiffness in some other direction. The following sections describe experimental and theoretical studies that have attempted to identify the best geometry for the Box Ribs.

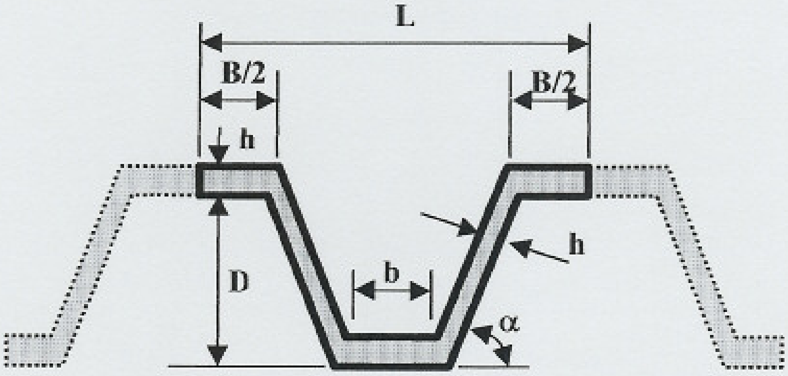


Fig 1 Geometry of Repeat Unit for Box Ribs

Experimental Investigation

A series of moulds were designed to rotationally mould the general type of box rib illustrated in Fig 1. Eight different configurations were used as detailed in Table 1. Each moulded panel was loaded using the configurations described above ie axial loading, bending with the loading parallel to the Box Ribs and bending with the loading perpendicular to the box ribs.

Shape	B(mm)	b(mm)	D(mm)	α
1	40	40	40	89
2	30	30	40	89
3	50	50	40	89
4	50	50	30	89
5	60	60	40	89
6	50	50	50	89
7	50	50	40	75
8	50	50	40	60

Table 1 Dimensions of Experimental Box Ribs

The stiffness (deflection per unit load) for each panel in tension shown in Fig 2. As would be expected, a flat plate gives the best performance in this type of loading but it may be seen that Box Rib Design #4 also gives a good performance. When the loading is in a bending mode, this Box Rib again performs very well. The short Box Rib depth in design #4 again gives the greatest stiffness when the loading is parallel to the Box Ribs (see Fig 3 and note enhancement in performance over flat plate), but it does not provide the best performance when the loading is perpendicular to the Box Ribs. However, in the latter case it gives a good average performance and is once again very much better than the flat plate (see Fig 4).

A detailed analysis of the experimental results, taking into account weight, etc led to the conclusion that Design #4 was best. It is not the optimum, but it gave the best overall performance for the shapes studied. The recommendation that the peaks and troughs of the Box Ribs should be equal and that the depth of the Box Ribs should be 60% of the width agreed well with the recommended design practice.

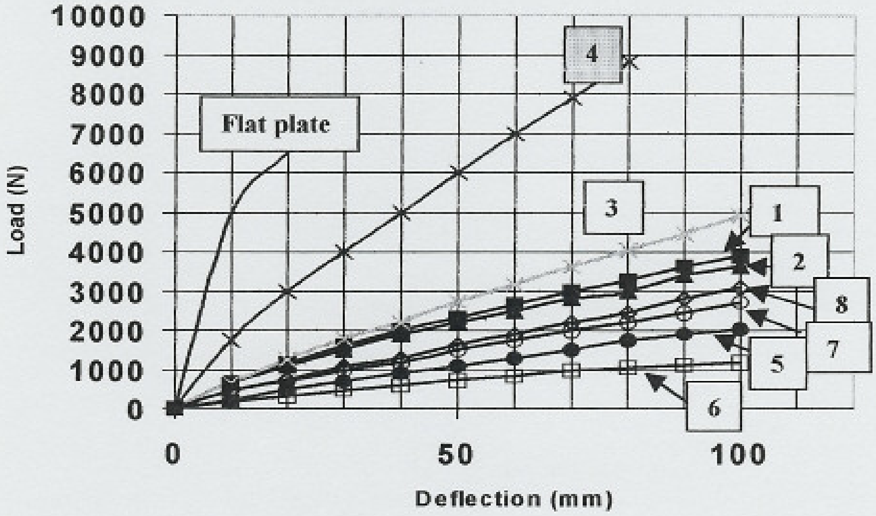


Fig 2 Deflection of Rotomoulded Box Ribs Loaded in Tension

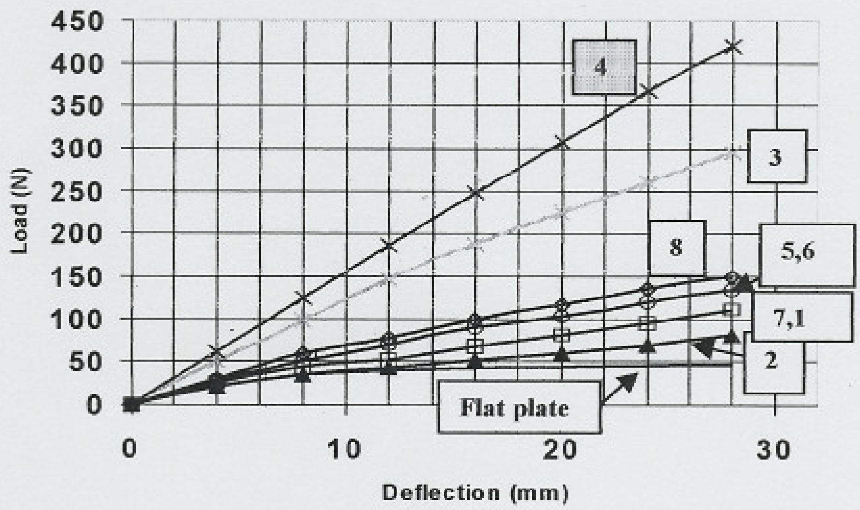


Fig 3 Deflection of Rotomoulded Sections Loaded in Parallel to Box Ribs

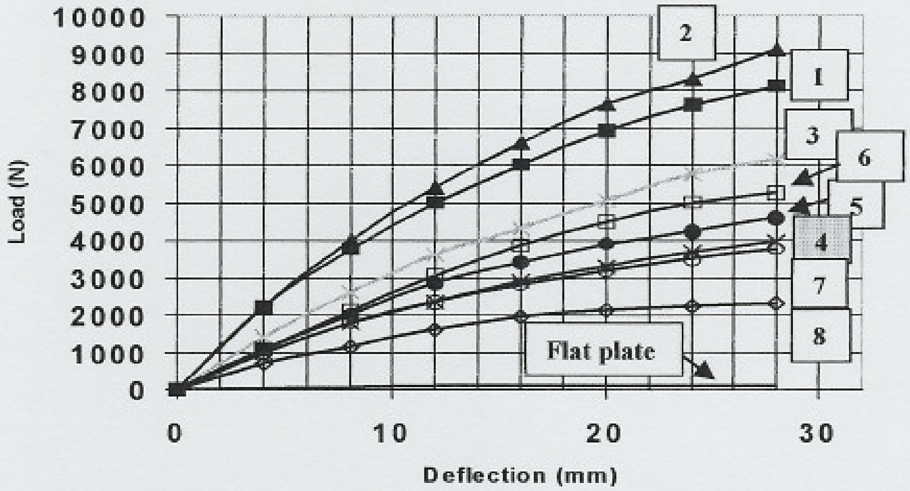


Fig 4 Deflection of Rotomoulded Sections Loaded in Flexure Perpendicular to Box Ribs

Theoretical Investigation

As indicated above, the experimental work, by its nature was not expected to lead to an optimum design. It is, however, relatively straightforward to analyse the section shown in Fig 1. Expressing the dimensions as a function of the thickness, h, of the moulding then the parameter that quantifies the transverse stiffness is the second moment of area, I. For bending in which the line of loading is perpendicular to the Box Ribs, the stiffness enhancement, q, relative to the flat sheet is given as:

$$q = \frac{1}{L}\{mh + \frac{12m}{h}(y - \frac{h}{2})^2 + \frac{12p}{h}[y - (nh + \frac{h}{2})]^2 + \frac{2nh}{\sin\alpha} + \frac{24n}{h\sin\alpha}(y - \frac{nh}{2})^2\}$$

where m=b/h, p=B/h and n=D/h and

$$y = h\{\frac{0.5m + pn + 0.5p + \frac{n^2}{\sin\alpha}}{(p + m + \frac{2n}{\tan\alpha})}\}$$

'y' is the distance of the centroid of the cross section from the bottom edge. Thus using this equation, the stiffness enhancement for any combinations of D, b, B, and L may be obtained.

Figure 5 shows the relative effects of some of the variables. For example, a typical design guideline is that b = 5h and D = 4h. Figure 5 shows that this gives a stiffness improvement of about 100 compared with a flat plate of the same thickness, h, and the same overall length, L. The stiffness enhancement is independent of the thickness, h. Figure 5 illustrates that there is no obvious optimum condition for the variables. To increase the transverse stiffness for the type of loading indicated, one should increase D/h, and the effect just keeps getting greater. One could, of course, consider dividing the stiffness enhancement by the area ratio of the new to the old section, to see if taking into account the amount of material used affects the predictions.

Stiffness of Corrugations as Function of Dimensions

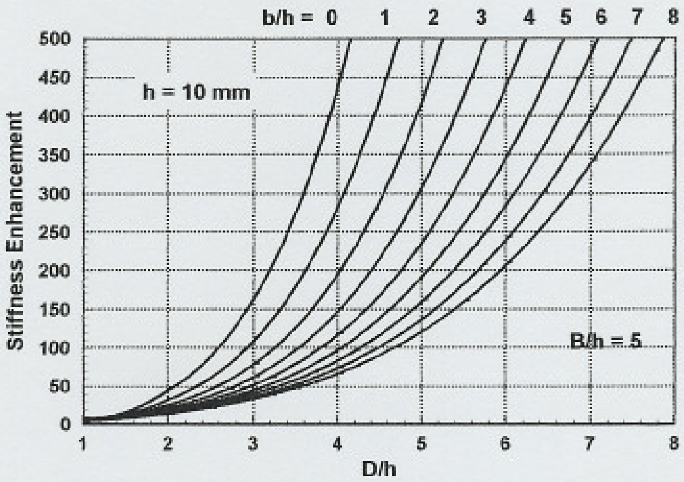


Fig 5 Stiffness Enhancement for Different Box Rib Geometries

Unfortunately, this does not help in identifying an optimum geometry. As the Box Rib depth increases, more material is used but the enhancement per unit weight does not go through an optimum condition.

Another approach is to consider the best combination of axial and transverse properties of the corrugated section. For axial loading of the Box Rib shown in Fig 1, it is desirable to have the axial movement as low as possible. Treating the Box Ribs as cantilevers, then it can be shown that the stiffness ratio, a, of the new to the old is given by:

If this is divided into the enhancement factor, q, then it is possible to observe a combined

$$a = 4\frac{n^3 h}{LSin^2\alpha}$$

parameter reflecting both axial and transverse properties. Fig 6 shows this parameter plotted against D/h. It is apparent that the best value of D/h is about 4. If lower values are used, then the performance is improved in the axial direction but weakened in the transverse direction. Conversely, if larger values of D/h are used, then the performance is improved in the transverse direction but at the expense of the axial direction.

Figure 6 is plotted for b/h = B/h = 5 and for a wall thickness of 10 mm ($\alpha = 85^\circ$ in all cases). It is interesting to note that the optimum value of D/h = 4 is what would be recommended by many design guidelines. However, it is important to note that this is only one of a number of "optimum" design configurations. The optimum value of D/h is essentially independent of the wall thickness, h, which is good news. However, it is very dependent on the combinations of b and B that are used. Figure 7 gives "optimum" values for D/h for a wide range of geometries. For example, if one wishes to use D/h = 4 with b/h = 5 (as per the usual Design Guidelines), then the preferred value of B/h is 4-5.

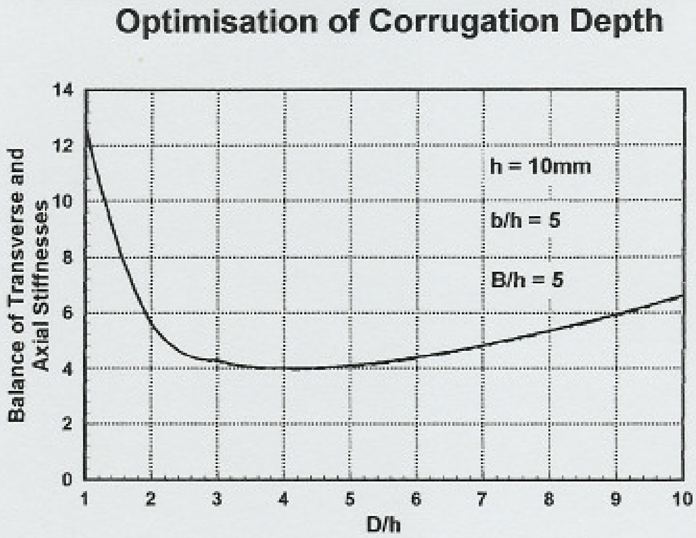


Fig 6 Optimising Ratio of Axial to Transverse Stiffness

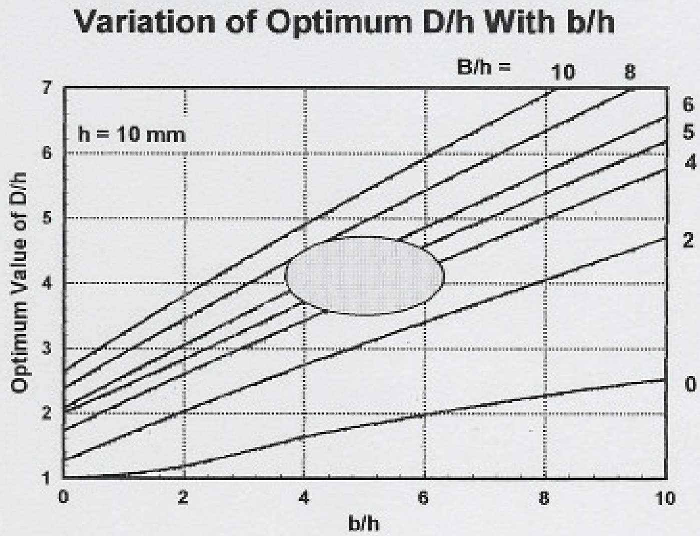


Fig 7 Optimum Design Window for Box Ribs

If one goes away from the shaded area in the centre of Figure 7, then B and b become quite different, and this lack of symmetry should suggest that conditions are moving away from the optimum. For short term loading, D/h should not exceed 10 if buckling is to be avoided. For long term loading, this critical ratio of D/h will decrease, and it is prudent not to exceed D/h = 6.

Thus the experimental work gave a best design as regularly spaced Box Ribs with the depth equal to 0.6 of this spacing. The theoretical analysis shows that this is slightly outside the "good design" window, suggesting that higher values of Box Rib depth are probably better. Designs in which 'B' is approximately equal to 'b' and the depth, D, is between 0.5 and 1 times 'b' will be the normal regions for good design practice. In practice it will depend to what extent there is axial loading applied to the moulded part.

Design Calculations for Rotationally Moulded Sandwich Foam Parts

The use of foamed materials in rotationally moulded parts is becoming very common. In some cases the foam is used to improve the thermal insulation properties of the part and in such cases, polyurethane foam is injected as a secondary operation or polyethylene foam is added during moulding. In other cases the foam is included to improve the stiffness to weight ratio of the part. If this is the objective then it is very important that there is a strong bond between the solid outer skin and the foamed inner layer. Otherwise the desired mechanical stiffening is not achieved. For this reason, if mechanical stiffening is the objective then polyethylene foam is usually preferred to polyurethane foam because the latter does not bond easily to polyethylene and extra surface treatment steps are needed to provide the necessary adhesion.

The following analysis looks at the use of sandwich foam structures in rotational moulding and considers some simple design calculations that can be performed to estimate the stiffening effect achieved by using the foam.

Background

The stiffness of a moulded part that is loaded in flexure (bending) is proportional to the product of the ***Second Moment of Area, I,*** and the ***Modulus, E,*** of the material. Designers of rotationally moulded parts must use shape very efficiently to increase the I value because the modulus of polyethylene is relatively low, and it decreases with time.

For a solid wall rotationally moulded part, the Second Moment of Area, *I*, is given by

$$I = (\text{width}) \times (\text{thickness})^3/12$$

It may be seen from this equation and Fig8 that a very effective way of increasing *I* is to increase the wall thickness of the moulded part. For example doubling the thickness will give 8 times (ie 2^3) greater stiffness. However, the weight of the part, and hence its cost, is doubled and the cycle time will be significantly increased.

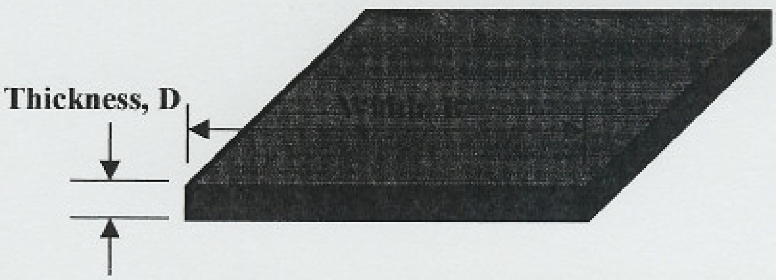
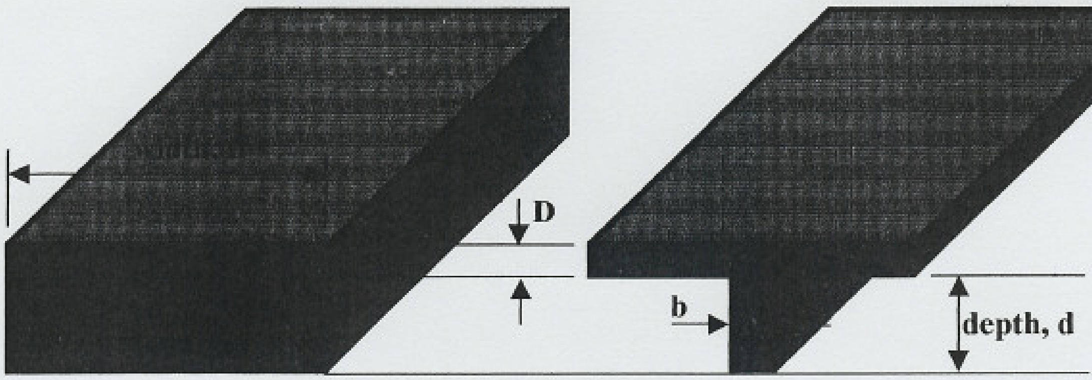


Fig 8 Factors used to calculate stiffness of solid rotomoulded part

Analysis of solid/foam cross-sections

The advantage to be gained by adding a foam layer may not be immediately apparent since the modulus of the foam is very much less than that of the solid plastic. In fact the use of the foam is like adding a stiffening rib to the plastic. This is illustrated in Fig 9.



(a) Solid/foam cross-section (b) Equivalent solid cross-section

Fig 9 Illustration that foam effectively acts as a rib

As it is difficult to analyse cross-sections that are made from different materials, the normal practice is to convert the solid/foam structure (on the left in Fig 9) into the equivalent cross-section shown on the right. The width of the "rib" is given by

$$b = (\frac{E_f}{E_s})B$$

where E_s is the modulus of the solid material and E_f is the modulus of the foamed material. Usually the modulus of the foam is not known, but if the skin and core are of the same base resin then the modular ratio can be expressed by a power law of the form:

$$\frac{E_f}{E_s} = (\frac{\rho_f}{\rho_s})^n$$

It has been found that for many plastics the index, n, is approximately 2, so this gives us a very convenient way of linking the thickness of the solid "rib" to the density of the foam, ρ.

In order to calculate the second moment of area, I, for the equivalent solid cross-section, we must determine the position of its centroid. If we take this to be at a distance y from the bottom edge then this distance is given as:

$$y = \frac{BD(d + \frac{D}{2}) + bd(\frac{d}{2})}{BD + bd}$$

The second moment of area, I, for the cross-section is then given as:

$$I = \frac{BD^3}{12} + BD(D + d - y - \frac{D}{2})^2 + \frac{bd^3}{12} + bd(y - \frac{d}{2})^2$$

It is interesting to explore the effects of different depths of the solid and foam layers for constant width, B, and length, L, into the page. Fig 10 shows how the I value varies and Fig 11 shows the weight of the corresponding cross-section. For example, a solid cross-section, with no foam layer, having a thickness of 6 mm will have an I value of $900 mm^4$ and a weight of 280.5g (assuming constant values of width as 50 mm and length as 1 m). If we add 6mm of foam to the solid layer then the weight goes up to 355g and the I value (proportional to stiffness) goes up to 1685mm^4. There is thus an increase of stiffness of 87%, and the stiffness per unit weight has increased by about 48%. The latter can be improved significantly if we reduce the thickness of the solid layer to, say 3mm, and increase the foam layer to 12mm. From Figs 10 and 11 we can see that the weight of this cross-section is 290.3g (approximately the same as the solid 6mm section) but the I value has now increased to 2503 mm^4. Thus the absolute value of the stiffness has increased by 178% and the stiffness per unit weight has increased by a similar amount. One can use Figs 10 and 11 to explore the effects of different combinations of solid skin layer and inner foam layer and in all cases the stiffness to weight ratio improves. This illustrates the benefits of using foaming of this type.

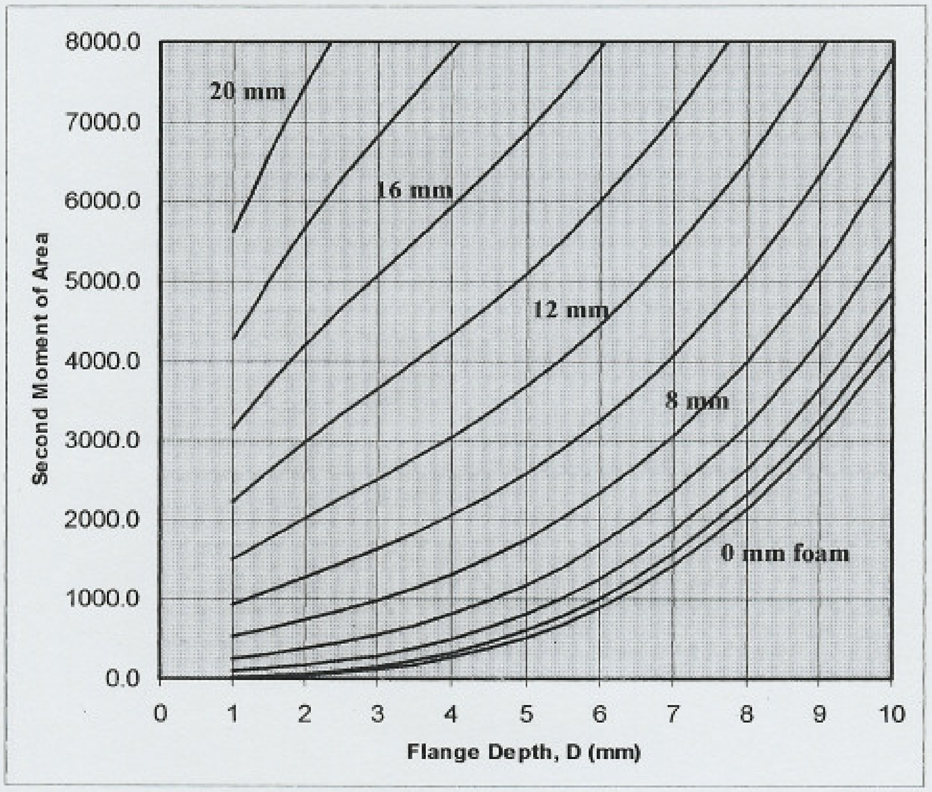


Fig 10 Variation of second moment of area as a function of the depths of the solid flange and the foam thickness

However, even more impressive gains can be made if the cross-section is enhanced with an inner solid layer. Such sandwich sections are extremely efficient at resisting bending stresses. In the context of rotational moulding, it does not matter if the solid/foam/solid cross-section completely fills the air space in the mould or if the inner solid layer contains the air space inside the part – the analysis is the same.

Analysis of solid/foam/solid cross-sections

If the foam layer is sandwiched between two solid skins then the analysis to calculate flexural stiffness is very similar to that shown above. Indeed in this case it is simpler because, due the symmetry as shown in Fig 12, the centroid of the cross-section is at half the overall depth.

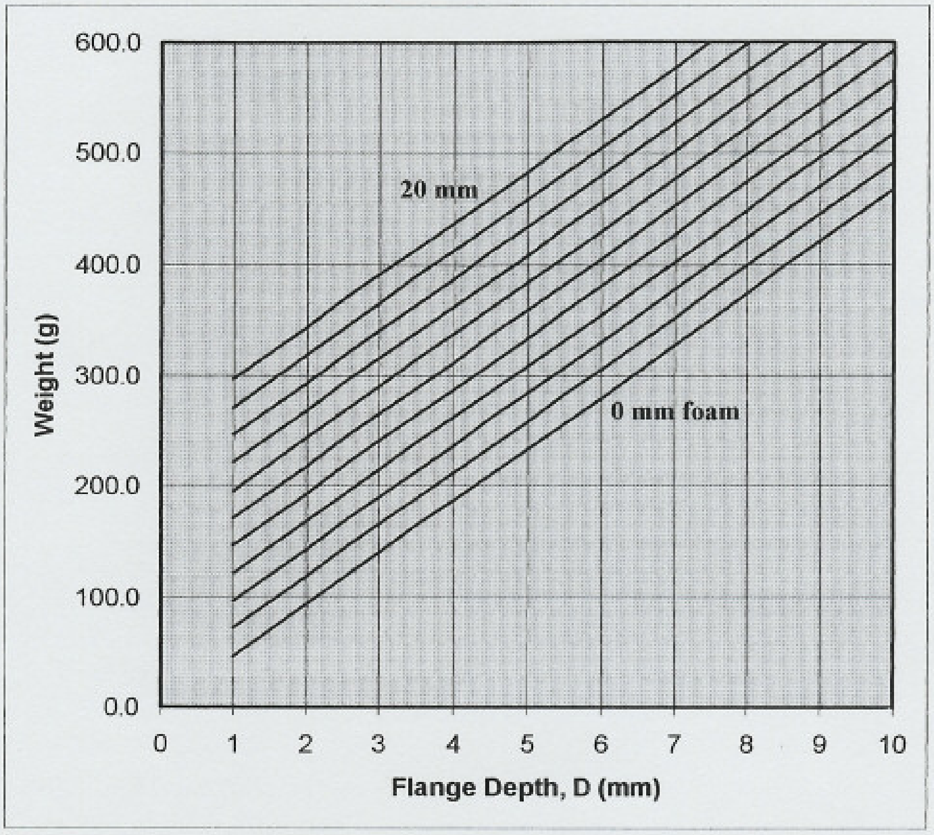


Fig 11 Variation of weight as a function of the depths of the solid flange and the foam thickness

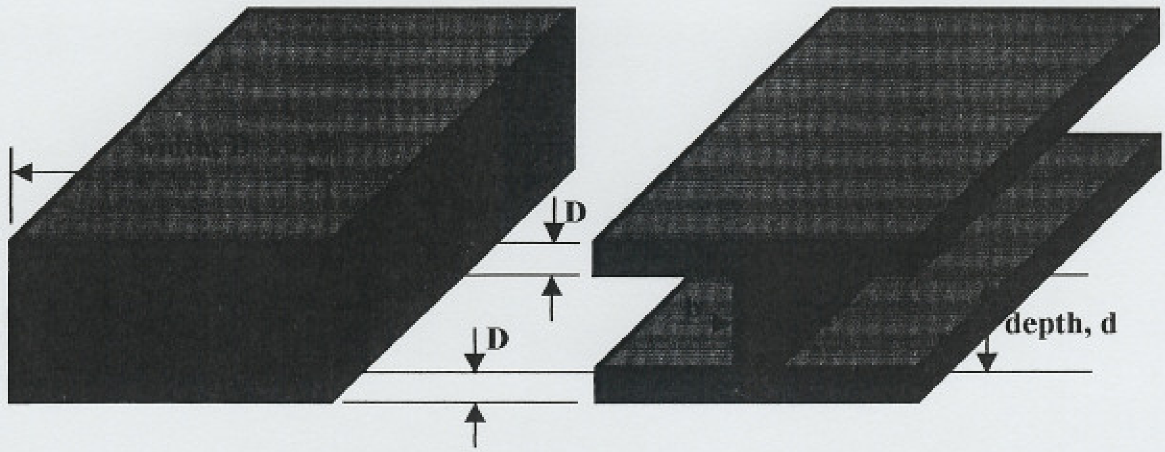
The Second Moment of Area, I, for this cross-section is given by:

$$I = \frac{B(D+d)^3}{12} - \frac{Bd^3}{12} + \frac{bd^3}{12}$$

where b may be determined from the foam density as above.

The variations of I and the weight of this type of structure, for different values of solid and foam thicknesses are given in Figs 13 and 14.

If we look at a similar example to above then it is possible to see the additional benefits of the second solid skin. For a solid 6mm cross-section, the second moment of area is 900mm^4 and the weight is 280.5g as previously (note that these values are obtained from the 3mm flange depth since the solid material is now split between the top and bottom surfaces). If we add 6 mm of foam between 3mm solid skins then the weight goes up to 355g as before but the second moment of area increases quite dramatically to 6364mm^4. Thus in absolute terms the stiffness of the cross-



(a) Solid/foam/solid cross-section (b) Equivalent solid cross-section

Fig 12 Foam effectively acts as a rib in sandwich cross-section

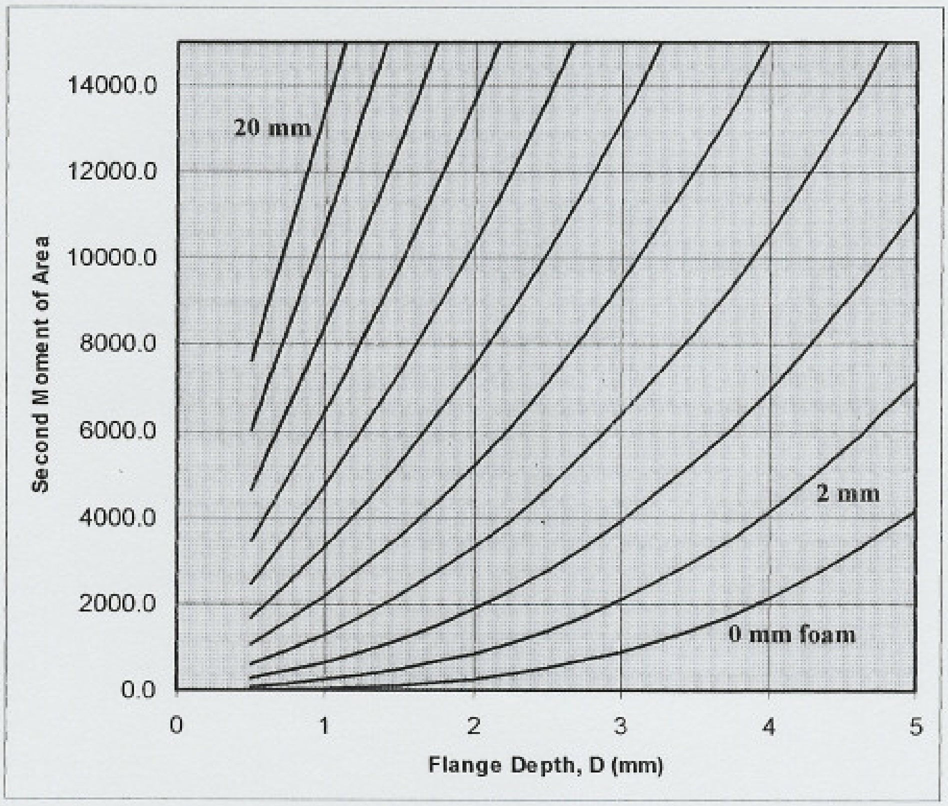


Fig 13 Variation of second moment of area for foam sandwich cross-section

section has gone up by a factor of 7 and the stiffness per unit weight has improved by 460%. If we take the other approach that we want to keep the weight approximately the same as the original 6mm solid cross-section, then we could use 1.5mm solid skins separated by 12mm of foam. The weight will be 290g as before but the second moment of area (stiffness) has gone up to 7377 mm^4, an increase of 719%.

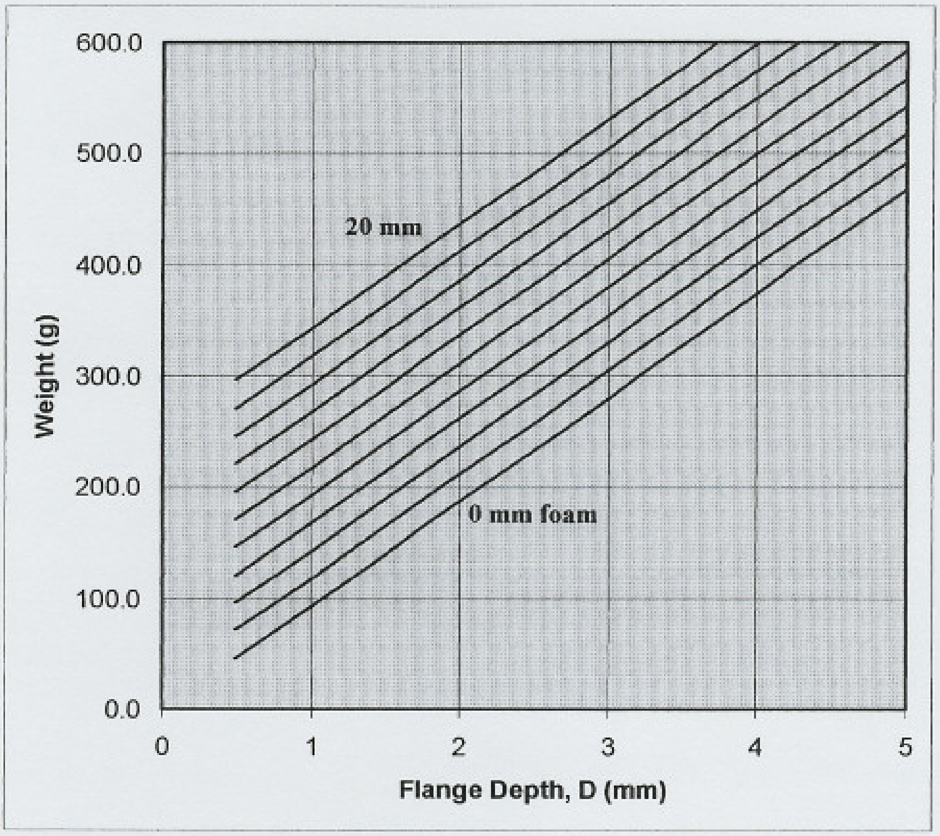


Fig 14 Variation of weight for foam sandwich cross-section

Conclusions

The experimental and theoretical analysis of the stiffness of box ribs produced by rotational moulding shows that there is no single optimum design. There are many factors that need to be considered, in particular the balance of transverse and axial loading and the ease of moulding. However, the analysis has shown that the generally accepted design guidelines of

Depth	4h
Root Width	5h
Top Width	5h

is close to the optimum for a good balance of axial and transverse stiffness.

It is clear that the use of foam sandwich cross-sections is an extremely efficient way of increasing flexural stiffness without any increase in weight. Indeed in many cases the moulding can be made much lighter and still have a greater stiffness.

It is also important to note that the use of a foam layer between two solid skins is much better than using a single solid layer with a single foam layer.

References

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